Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2012

Name:______________________________

Instructor: Migliore

Exam I - Answers

February 9, 2012

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

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Place an $\times$ through your answer to each problem.

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Multiple Choice

1. (5 pts.) Let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c, d, e\}$, and $B = \{c, d, e, f, g\}$. Find $(A \cap B)' \cap A$.

(a) $\{h\}$ (b) $\{a, b, f, g, h\}$ (c) $\{a, b\}$

(d) $\emptyset$ (e) $\{c, d, e\}$

Solution: $A \cap B = \{c, d, e\}$, so $(A \cap B)' = \{a, b, f, g, h\}$. Then $(A \cap B)' \cap A = \{a, b\}$.

2. (5 pts.) Let

$$\begin{aligned} U &= \{\text{all people}\} \\ A &= \{\text{people who have been in Argentina}\} \\ B &= \{\text{people who have been in Bolivia}\} \end{aligned}$$

Which of the following represents in words the set $(A \cap B)'$? [Hint: First decide what $A \cap B$ means, then decide what its complement is.]

(a) People who have not been in **either** Argentina **or** Bolivia.

(b) People who have **either** been in Argentina but not Bolivia, **or** in Bolivia but not Argentina.

(c) People who have been in Argentina and Bolivia.

(d) People who have never been in South America.

(e) People who have not been in **both** Argentina **and** Bolivia.

Solution: $A \cap B$ is the set of people who have been to both Argentina and Bolivia. $(A \cap B)'$ is the set of people who have not been to BOTH Argentina and Bolivia. They may have been to one, but not both.

3. (5 pts.) In the town of Hatsboro, there are 5,400 people. Of these, 3,000 own a red hat (and possibly other hats) and 3,000 own a blue hat (and possibly other hats). Among these, 2,500 own **both** a red hat and a blue hat. How many do not own either a red hat or a blue hat?

(a) 1,900 (b) 3,500 (c) 600 (d) 3,100 (e) 2,900

Solution: Let R be the set of people who own a red hat and B the set of people who own a blue hat. We have $n(R) = 3,000$, $n(B) = 3,000$ and $n(R \cap B) = 2,500$. Also, $n(U) = 5,400$. Now,

$$n(R \cup B) = n(R) + n(B) - n(R \cap B)$$

(the inclusion-exclusion principle), so $n(R \cup B) = 3,000 + 3,000 - 2,500 = 3,500$. The question is asking for the number of elements in $(R \cup B)'$, which is $5,400 - 3,500 = 1,900$.

4. (5 pts.) Five faculty members and six students are planning to get in line for tickets to see a Shakespeare production. It's agreed that the students will all be in front of the faculty members. In how many different orders can they line up for their tickets?

(a) $6! + 5!$ (b) $11!$ (c) $6! \cdot 5!$

(d) $2 \cdot 6! \cdot 5!$ (e) $2(6! + 5!)$

Solution: There are 6! possible orders for the students, and for each of these there are 5! possible orders for the faculty. Thus there are $6! \cdot 5!$ total possible orders.

5. (5 pts.) A club consisting of ten men and twelve women decide to make a brochure to attract new members. On the cover of the brochure, they want to have a picture of two men and two women from the club. How many pictures are possible (taking into account the order in which they line up for the picture)? [Hint: there are at least two ways to do this, so if you don't see your answer in terms of C's and P's, compute the numerical value just in case it matches one of these.]

(a) $[C(10,2) \cdot C(12,2)]! = 2,970$

(b) $C(10,2) \cdot C(12,2) \cdot 4! = 71,280$

(c) $P(10,2) \cdot P(12,2) = 11,880$

(d) $[C(10,2) + C(12,2)] \cdot 4! = 2,664$

(e) $P(10,2) \cdot P(12,2) \cdot 4! = 285,120$

Solution 1: First decide which two men and which two women will be on the cover, without worrying about order. There are $C(10,2) \cdot C(12,2)$ ways to do this. Now you have four people. There are $P(4,4) = 4!$ ways to order them. So the answer is $C(10,2) \cdot C(12,2) \cdot 4! = 71,280$.

Solution 2: Decide which of the four positions will be men (and the other two will be women). There are $C(4,2)$ ways to do this. Now you know which two spots are for men, and there are $P(10,2)$ ways to fill these spots. And you know which two spots are for women, and there are $P(12,2)$ ways to fill these spots. So the answer is $C(4,2) \cdot P(10,2) \cdot P(12,2) = 71,280$.

6. (5 pts.) Suppose you know that $C(n,3) = 56$. Find n. [Hint: one quick way to do this is with Pascal's triangle.]

(a) 10 (b) 7 (c) 9 (d) 8 (e) 11

Solution:

$$\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\
1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1
\end{array}$$

So $56 = C(8,3)$.

7. (5 pts.) To order a pizza, you have to first choose a sauce and then choose toppings. There are three kinds of sauces (red, white and green) and six kinds of toppings (mushroom, pepperoni, sausage, green pepper, artichoke and seaweed). You must choose one of the three sauces, and **at least one** topping. How many different pizzas can be created?

(a) 191 (b) 189 (c) 378 (d) 383 (e) 18

Solution: You choose one sauce (3 choices) and some subset of the six toppings **except the empty set**. There are $2^6 - 1 = 63$ such subsets. So the answer is $3 \cdot 63 = 189$.

8. (5 pts.) Suppose that an experiment consists of tossing a coin 10 times and recording the sequence of heads and tails. How many different outcomes have exactly 2 heads and 8 tails?

(a) 80,640 (b) 1024 (c) 90 (d) 45 (e) 16

Solution: You choose which two of the ten tosses are heads. There are $C(10, 2) = 45$ ways to do this.

9. (5 pts.) Calculate the value of $C(201, 199)$.

(a) 40,400 (b) 402 (c) 39,999

(d) 200 (e) 20,100

Solution: $C(201, 199) = C(201, 201 - 199) = C(201, 2) = \dfrac{201 \cdot 200}{2} = 20,100.$

10. (5 pts.) There are 100 Senators in the U.S. Senate, two from each of the 50 states. A committee of six Senators is to be formed, such that no two are from the same state. In how many ways can this be done?

(a) $C(100, 6) - 2^6$ (b) $C(50, 6) \cdot 2^6$ (c) $C(50, 6) \cdot 6^2$

(d) $\dfrac{C(100, 6)}{50}$ (e) $C(50, 6) \cdot 2^{50}$

Solution: First choose which six states will be represented on the committee. There are $C(50, 6)$ ways to do this. So now you have six states, and for each you have two choices for the senator. There are $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$ ways to do this. So the final answer is

$$C(50, 6) \cdot 2^6.$$

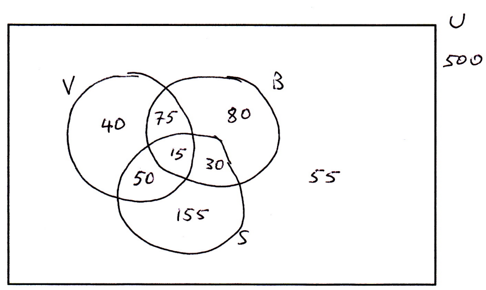
Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) All of the 500 students at State University are required to participate in at least one sport (varsity or intramural). Suppose 180 play volleyball, 200 play basketball, 250 play softball, 50 play volleyball and softball but not basketball, 75 play volleyball and basketball but not softball, 40 play only volleyball, 80 play only basketball.

(a) Draw and label a Venn Diagram representing the above information. Be sure to put a number in all relevant regions of your diagram.

Solution: Let V be the set of volleyball players, B the set of basketball players and S the set of softball players. Let U be the universal set of all 500 students.



(b) How many play only softball?

Solution: 155

(c) How many do not play any of the three sports?

Solution: 55

12. (10 pts.) A family has nine cats and three dogs. They want to choose four out of these twelve pets to take with them on a vacation.

(a) In how many ways can this be done?

Solution: There are 12 pets and they want to choose 4 of them. There are $C(12, 4) = 495$ ways.

(b) In how many ways can this be done if they have to take at least one dog?

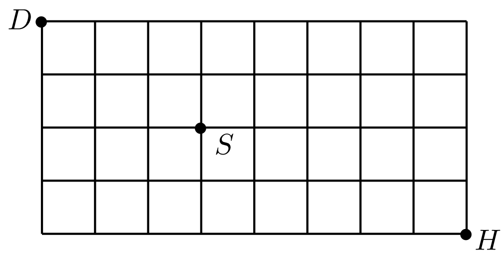
Solution: Ways to take exactly one dog (and three cats): $C(3, 1) \cdot C(9, 3) = 3 \cdot 84 = 252$.

Ways to take exactly two dogs (and two cats): $C(3, 2) \cdot C(9, 2) = 3 \cdot 36 = 108$.

Ways to take exactly three dogs (and one cat): $C(3, 3) \cdot C(9, 1) = 1 \cdot 9 = 9$.

So all together, there are $252 + 108 + 9 = 369$ ways.

13. (10 pts.) All three parts of this problem refer to the following city map. For this problem, you may leave your answer in terms of mixtures of combinations and permutations (i.e. $C(n,r)$ and $P(n,r)$ for appropriate n and r) if you like.



(a) On Saturday, Dave starts at the point marked D and needs to get home, marked H. If he has to do it in as few blocks as possible (12), in how many ways can it be done? (Ignore the "S" at this point.)

Solution: There are 12 blocks in the route, and he has to choose which 4 will be South. There are $C(12,4) = 495$ ways.

(b) On Sunday, Dave is at point D and wants to watch the Superbowl at Sam's house, at point S. However, he needs to go home first to get his glasses. If he wants to make the trip from D to H and then from H to S in as few total blocks as possible (19), in how many ways can it be done.

Solution: $C(12,4) \cdot C(7,2) = 495 \cdot 21 = 10,395$.

(c) On Monday, Dave is at point D and wants to go home. However, he owes Sam \$100 from losing a bet on the Superbowl, so he wants to choose a 12-block route that does **not** pass through point S. In how many ways can this be done?

Solution: Count the total number of ways to get from D to H (from part (a)) and subtract the number that go through S.

$C(12,4) - C(5,2) \cdot C(7,2) = 495 - 10 \cdot 21 = 495 - 210 = 285$.

14. (10 pts.) An urn contains 12 numbered balls, of which 5 are red and 7 are blue. Bob has to select 4 of these balls. Please give numerical answers.

(a) In how many ways can this be done, independent of the color of the balls?

Solution: $C(12, 4) = 495$.

(b) How many samples contain only blue balls?

Solution: $C(7, 4) = 35$.

(c) How many samples contain at least 3 red balls?

Solution: Ways to get exactly 3 red: $C(5, 3) \cdot C(7, 1) = 70$.

Ways to get exactly 4 red: $C(5, 4) = 5$.

So total number of ways to get at least 3 red balls is $70 + 5 = 75$.

15. (10 pts.) Six married couples are going to be in a group picture, all lined up in a row.

(a) In how many ways can the 12 people line up? You can give your answer using either C, P or factorial notation, or you can give a numerical answer.

Solution: $P(12, 12) = 12! = 479,001,600$.

(b) In how many ways can they line up if everyone has to be standing next to their spouse? You can give your answer using either C, P or factorial notation, or you can give a numerical answer.

Solution: There are six couples. First decide the order of the couples. There are $P(6, 6) = 6! = 720$ such ways. Now we know the order of the couples. For each couple, there are 2 ways they can line up, so all together we have $6! \cdot 2^6 = 46,080$ possible lineups.

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